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Solutions of Exercise Sheet 9: Multiresolution Analysis

Exercise 1 *(Multiresolution Analysis)*

We obtain $\psi_{m,k}(x) = \sum_{j\in\mathbb{Z}} g_j \varphi_{m-1,2k+j}(x)$, which proves the first statement. Using this result and the relation $\sum_{k\in\mathbb{Z}} h_{k+2j}\bar{h}_k = \delta_{0,j}$ we get $\langle\psi_{m,k}|\psi_{n,l}\rangle = \delta_{m,n}\delta_{l,k}$, so that $(\psi_{m,k})_{m,k\in\mathbb{Z}}$ is an orthonormal system. Furthermore, $\langle\psi_{0,k}|\varphi_{0,l}\rangle = 0$ implies that all $\psi_{0,k} \in V_{-1}$ are elements of W_0. To verify completeness of the orthonormal system, it is sufficient to show Parseval's identity $\|x\|^2 = \sum_{b\in B} |\langle x|\rangle b|^2$ with $B := \{\varphi_{0,k}, \psi_{0,k}|k \in \mathbb{Z}\}$ for $x = \varphi_{-1,0}$ which completes the proof of the other statements.

Exercise 2 *(Laplace Transform)*

We can easily verify the linearity of the Laplace transform and $\mathcal{L}(\dot{f})(s) = sF(s) - f_0$, so that we obtain $F(s) = f_0/(s+\lambda)$ and finally the well known solution $f(t) = \mathcal{L}^{-1}(f_0/(s+\lambda))(t) = f_0 \exp(-\lambda t)$ of the first-order linear Cauchy problem.