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High Fidelity Algorithmics Group
CS Dept. Stanford University

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CS448J: CASVC 2015 @ Stanford
Exercise Sheet 9: Multiresolution Analysis

Exercise 1 *(Multiresolution Analysis, 2 + 4 + 4 = 10 Points)*

Let $((V_m)_{m\in\mathbb{Z}}, \varphi)$ a multiresolution analysis with scaling coefficients $h \in \ell^1(\mathbb{Z})$. With $g_k := (-1)^k \overline{h}_{1-k}$ we define $\psi \in V_{-1}$ with $\psi(x) := \sum_{k\in\mathbb{Z}} g_k \varphi_{-1,k}(x)$. For the functions

$$\psi_{m,k}(x) := 2^{-m/2}\psi(2^{-m}x - k), \quad m, k \in \mathbb{Z},$$

prove the following statements:

1. $\psi_{m,k} = \sum_{j\in\mathbb{Z}} g_j \varphi_{m-1,2k+j}$,
2. $(\psi_{m,k})_{k\in\mathbb{Z}}$ is a Hilbert basis of W_m,
3. $(\psi_{m,k})_{m,k\in\mathbb{Z}}$ is a Hilbert basis of $L^2(\mathbb{R}) = \overline{\bigoplus_{m\in\mathbb{Z}} W_m}$.

Exercise 2 *(Laplace Transform, 6 Points)*

Given a function, $f : [0, \infty) \to \mathbb{C}$, its Laplace transform is defined by

$$F(s) = \mathcal{L}(f)(s) := \int_0^\infty f(t)\exp(-st)\,\mathrm{d}t$$

for $s \in \mathbb{C}$.[1] Moreover, the inverse Laplace transform is given by the Bromwich integral

$$f(t) = \mathcal{L}^{-1}(F)(t) := \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} F(s)\exp(st)\,\mathrm{d}s,$$

in which $\gamma \in \mathbb{R}$ enforces that the contour path of the integration is in the region of convergence of F. Determine the solution of the ordinary first-order linear Cauchy problem

$$\dot{f}(t) + \lambda f(t) = 0$$

with $f(0) = f_0$ by applying a Laplace transform to the differential equation, solving the resulting algebraic equation, and finally performing a back projection into the original domain.

Notes

- The Homework is due by 10:30am on Dec. 1. Written solutions should be handed in before the lecture. Programming assignments must be submitted by email to your tutor David Hyde <dabh@stanford.edu>.
- In case you have any questions about the assignments, please contact your tutor David Hyde <dabh@stanford.edu> or the instructor Prof. Dominik L. Michels <michels@cs.stanford.edu> directly via email.
- Office hours are every Friday, 10-12 in 208/209 Gates CS Bldg. or by appointment.
- The university expects both faculty and students to respect and follow Stanford's Honor Code; see https://communitystandards.stanford.edu/.

[1] We obtain its natural connection to the Fourier transform by setting $s := 2\pi i\omega$.