Prof. Dominik L. Michels
David Hyde

High Fidelity Algorithmics Group
CS Dept. Stanford University

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CS448J: CASVC 2015 @ Stanford
Exercise Sheet 6: Fourier Analysis

Exercise 1 *(Fourier Analysis, 2 + 2 + 2 + 2 + 2 + 2 = 12 Points)*

For a given function $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, show the following properties of its Fourier transform $\mathcal{F}(f)$.

1. For $\bar{t} \in \mathbb{R}$ and the $\bar{t}$-translation $f_{\bar{t}} := f(t - \bar{t})$ holds
$$\mathcal{F}(f_{\bar{t}})(\omega) = \exp(-2\pi i\omega\bar{t})\mathcal{F}(f)(\omega).$$
2. For $\bar{\omega} \in \mathbb{R}$ and the $\bar{\omega}$-modulation $f^{\bar{\omega}} := \exp(-2\pi i\bar{\omega}t)f(t)$ holds
$$\mathcal{F}(f^{\bar{\omega}})(\omega) = \mathcal{F}(f)(\omega + \bar{\omega}).$$
3. Let f be differentiable with $f' := \mathrm{d}_t f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$. Furthermore, $\lim_{t\to\pm\infty} f(t) = 0$. Then holds
$$\mathcal{F}(f')(\omega) = 2\pi i\omega\mathcal{F}(f)(\omega).$$
4. Let f be n-times continuously differentiable with derivatives $f', \ldots, f^{(n)} \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$. Determine the relation between the Fourier transforms of $f', \ldots f^{(n)}$ and the Fourier transform of f.
5. Let $\mathcal{F}(f)$ be differentiable and $g(t) := tf(t)$. Then holds for $\mathcal{F}(f)' := \mathrm{d}_\omega \mathcal{F}(f)$
$$\mathcal{F}(f)'(\omega) = -2\pi i\mathcal{F}(g)(\omega).$$
6. For $s \in \mathbb{R} \setminus \{0\}$, the s-scaled version $f_{|s} : t \mapsto f(t/s)$ of f is also an element of $L^2(\mathbb{R})$ and it holds
$$\mathcal{F}(f_{|s})(\omega) = |s|\mathcal{F}(f)(\omega s).$$

Notes

- The Homework is due by 10:30am on Nov. 10. Written solutions should be handed in before the lecture. Programming assignments must be submitted by email to your tutor David Hyde <dabh@stanford.edu>.
- In case you have any questions about the assignments, please contact your tutor David Hyde <dabh@stanford.edu> or the instructor Prof. Dominik L. Michels <michels@cs.stanford.edu> directly via email.
- Office hours are every Friday, 10-12 in 208/209 Gates CS Bldg. or by appointment.
- The university expects both faculty and students to respect and follow Stanford's Honor Code; see https://communitystandards.stanford.edu/.