Prof. Dominik L. Michels
David Hyde

High Fidelity Algorithmics Group
CS Dept. Stanford University

Autumn 2015

CS448J: CASVC 2015 @ Stanford
Solutions of Exercise Sheet 5: Heat Equation

Exercise 1 *(Heat Equation)*

For a given function $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x)$, we obtain from its Taylor series expansions around Δx and $-\Delta x$,

$$f'(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x} + \mathcal{O}(\Delta x)$$

and

$$f''(x) = \frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x^2} + \mathcal{O}\left(\Delta x^2\right).$$

The componentwise application of these expressions onto the diffusion equation at time $t + \Delta t$ and the simplications $\Delta x = \Delta y$ and $\tau := -\Delta t / \Delta x^2$ lead to a linear system defined by

$$\begin{aligned} u(x, y, t) = (1 - 4\tau)\ & u(x, y, t + \Delta t) \\ +\tau\ & u(x + \Delta x, y, t + \Delta t) \\ +\tau\ & u(x - \Delta x, y, t + \Delta t) \\ +\tau\ & u(x, y + \Delta y, t + \Delta t) \\ +\tau\ & u(x, y - \Delta y, t + \Delta t). \end{aligned}$$

The initial temperatures can be extracted from an input image, e.g. from the Stanford logo. The results are shown below.