Exercise 1 (Heat Equation, 15 Points)

Let

\[ G = \{0, \Delta x, \ldots, (n-1)\Delta x\} \times \{0, \Delta y, \ldots, (m-1)\Delta y\} \]

be a \((n \times m)\)-grid and let \(u(x, y, t)\) be the temperature at \((x, y) \in G\) at time \(t\). The temporal evolution of the temperature is described by the (two-dimensional partial differential) heat equation

\[ \partial_t u(x, y, t) = \nabla \cdot J = \partial_x J_x + \partial_y J_y, \]

in which \(J = (J_x, J_y)^T\) denotes the rate of the heat flow. If the heat flow is purely resulting from a diffusion effect, we obtain \(J = \nabla u\), which simplifies the heat equation to the diffusion equation

\[ \partial_t u(x, y, t) = \Delta u = \partial_x^2 u(x, y, t) + \partial_y^2 u(x, y, t). \]

Implement a diffusion equation solver using the finite difference method and compute a simple diffusion process. Extract your initial temperature values from a simple image.

Notes

- The Homework is due by 10:30am on Nov. 3. Written solutions should be handed in before the lecture. Programming assignments must be submitted by email to your tutor David Hyde <dabh@stanford.edu>.

- In case you have any questions about the assignments, please contact your tutor David Hyde <dabh@stanford.edu> or the instructor Prof. Dominik L. Michels <michels@cs.stanford.edu> directly via email.

- Office hours are every Friday, 10-12 in 208/209 Gates CS Bldg. or by appointment.

- The university expects both faculty and students to respect and follow Stanford’s Honor Code; see https://communitystandards.stanford.edu/.