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High Fidelity Algorithmics Group
CS Dept. Stanford University

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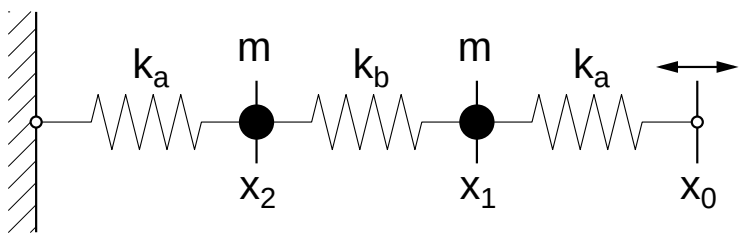
CS448J: CASVC 2015 @ Stanford
Exercise Sheet 4: Fermi-Pasta-Ulam Problem

Exercise 1 *(Fermi-Pasta-Ulam Problem, 10 Points)*

Simulate the dynamics of a simple Fermi-Pasta-Ulam problem which is described by the equations of motion

$$\ddot{\boldsymbol{x}} + \frac{1}{m}\begin{bmatrix} k_a + k_b & -k_b \\ -k_b & k_a + k_b \end{bmatrix} \boldsymbol{x} = \frac{1}{m}\begin{bmatrix} k_a \hat{x}_0 \cos(\omega t) \\ 0 \end{bmatrix},$$

in which k_b denotes the stiffness of the middle spring and k_a denotes the stiffness of both other springs. At initial time the system resides in an equilibrium state and is then excited by an external force acting sinusoidally with a frequency of ω and an amplitude of $\hat{x}_0$.



The ratio κ of the extremal eigenvalues of the coefficient matrix is a measure of the stiffness of the system. Because of $\kappa = 1 + 2k_b/k_a$ the stiffness of the system is large for $k_b \gg k_a$.

Plot the trajectories $x_1(t)$ and the phase portraits $x_2(x_1)$, and $v_1(x_1)$ of this Fermi-Pasta-Ulam problem for its solution computed with the Gautschi type exponential time integrator and the variational implicit-explicit integrator. Make use of the parameter setting $m = 1.0$, $k_a = 10^2$, $k_b = 10^{10}$, $\hat{x}_0 = 1.0$, and $\omega = \sqrt{k_a/m} - 10^{-10}$.

Interpret your results using a comparison with the phase portraits determined by the analytical solution which is given by $\boldsymbol{x}(t) = \hat{\boldsymbol{x}} \cos(\omega t)$ with the amplitude

$$\hat{\boldsymbol{x}} := \frac{k_a}{m}\hat{x}_0 \left(\omega_1^2 - \omega^2\right)^{-1} \left(\omega_2^2 - \omega^2\right)^{-1} \begin{bmatrix} \omega_3^2 - \omega^2 \\ \omega_3^2 - \omega_1^2 \end{bmatrix},$$

and the natural frequencies of $\omega_1^2 := k_a/m$, $\omega_2^2 := (k_a + 2k_b)/m$, and $\omega_3^2 := (k_a + k_b)/m$.

Notes

- The Homework is due by 10:30am on Oct. 27. Written solutions should be handed in before the lecture. Programming assignments must be submitted by email to your tutor David Hyde <dabh@stanford.edu>.
- In case you have any questions about the assignments, please contact your tutor David Hyde <dabh@stanford.edu> or the instructor Prof. Dominik L. Michels <michels@cs.stanford.edu> directly via email.
- Office hours are every Friday, 10-12 in 208/209 Gates CS Bldg. or by appointment.
- The university expects both faculty and students to respect and follow Stanford's Honor Code; see https://communitystandards.stanford.edu/.