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Solutions of Exercise Sheet 1: Fundamental Lemma

Exercise 3 *(Fundamental Lemma of Calculus of Variations)*

The function α_τ with

$$\alpha_\tau(t) = (\tau - t + \epsilon)(t - \tau + \epsilon)^3$$

for $t \in (\tau - \epsilon, \tau + \epsilon)$ and $\alpha_\tau(t) = 0$ for $[t_i, t_f] \setminus (\tau - \epsilon, \tau + \epsilon)$ is twice continuously differentiable.

We assume that there would be a $\tau \in (t_i, t_f)$ with $f(\tau) \neq 0$ and can easily construct a contradiction: because of f's continuity, $f(t) \neq 0$ also holds for all $t \in (\tau - \epsilon, \tau + \epsilon)$ and hence

$$\int_{t_i}^{t_f} f(t)\alpha(t)\,\mathrm{d}t \neq 0$$

holds for $\alpha = \alpha_\tau$.