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High Fidelity Algorithmics Group
CS Dept. Stanford University

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CS448J: CASVC 2015 @ Stanford
Exercise Sheet 1: Variational Principles

Exercise 1 *(Double Pendulum with Lagrange, 5 + 5 + 2 + 2 = 14 Points)*

Consider a double pendulum in a gravitational field/frictionless environment as illustrated in Fig. 1.

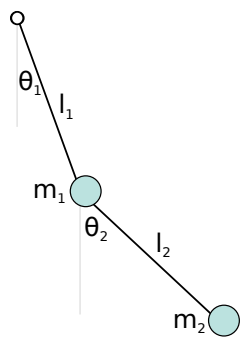

Figure 1: Double pendulum with particle masses m_1, m_2 and stiff rods with lengths l_1, l_2.

1. Derive the second order equations of motion using appropriate generalized coordinates and the Euler-Lagrange formalism. (Hint: A solution sketch can be found on the lecture slides.)
2. Implement a simple forward Euler scheme to solve the previously derived equations of motion numerically.
3. Plot phase space diagrams and visualize the motion dynamically.
4. Run different experiments with varying temporal step sizes and discuss your results.

Exercise 2 *(Double Pendulum with Hamilton, 5 + 1 = 6 Points)*

1. Derive the equivalent first order equations of motion of the double pendulum using the Hamiltonian formalism.
2. Show their equivalence to the second order equations derived in the previous exercise.

Exercise 3 *(*Fundamental Lemma of Calculus of Variations, 5 Bonus Points)*

Prove that for a continuous function $f : \mathbb{R} \supseteq [t_i, t_f] \ni t \mapsto f(t) \in \mathbb{R}$ with

$$\int_{t_i}^{t_f} f(t)\alpha(t)\,\mathrm{d}t = 0$$

for all twice continuously differentiable functions $\alpha : \mathbb{R} \supseteq [t_i, t_f] \ni t \mapsto a(t) \in \mathbb{R}$ with $\alpha(t_i) = \alpha(t_f) = 0$, holds $f(t) = 0$ for all $t \in [t_i, t_f]$.

Notes

- The Homework is due by 10:30am on Oct. 6. Written solutions should be handed in before the lecture. Programming assignments must be submitted by email to your tutor David Hyde <dabh@stanford.edu>.
- In case you have any questions about the assignments, please contact your tutor David Hyde <dabh@stanford.edu> or the instructor Prof. Dominik L. Michels <michels@cs.stanford.edu> directly via email.
- Office hours are every Friday, 10-12 in 208/209 Gates CS Bldg. or by appointment.
- The university expects both faculty and students to respect and follow Stanford's Honor Code; see https://communitystandards.stanford.edu/.